ON THE ESTIMATION OF ELASTIC SCATTERING CROSS SECTIONS OF GAMMA RAYS FROM DIFFERENT ELEMENTS

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ABSTRACT A method for the estimation of elastic scattering cross sections of low energy gamma rays from different elements at different scattering angles is described It [illegible] of the exact calculations of Brown and Mayers for mercury and the experimental data on Z dependence The estimated values show a good agreement with the available experimental data

INTRODUCTION

Rayleigh scattering from atomic electrons and Thomson scattering from nuclear charge are the only processes which make any significant contributions to the elastic scattering of low energy gamma rays While Thomson scattering from different elements can be accurately calculated, the exact calculations of Rayleigh scattering are available only for K-electrons of mercury. However, various approximate form factor calculations have been proposed by many workers to estimate Rayleigh scattering from different elements. Unfortunately, these form factor calculations have been found to be in large errors. In a recent communication, we suggested that our experimental studies of Z dependence of elastic scattering of gamma rays can be combined with the refined calculations of Brown and Mayers (1957) to give reliable estimates of elastic scattering cross sections of gamma rays from different elements at various scattering angles and gamma ray energies (Anand *et al.*, 1964) In the following sections we outline the method of estimation and compare the results so obtained with the available experimental data

METHOD OF ESTIMATION

The refined calculations of Brown and Mayers (1957) for Rayleigh scattering are given in terms of K-shell scattering amplitudes for mercury with and without polarization change The cross section for Rayleigh scattering is given in the form :

$$(d\sigma/d\Omega)_k = r_0^2[\,|a_{1k}+ib_{1k}|^2+|a_{2k}+ib_{2k}|^2] \qquad \dots \quad (1)$$

where subscripts $1k$ and $2k$ denote the K-shell amplitudes with no polarization change and with polarization change respectively, and r_0 is the classical electron radius. These amplitudes are complex quantities; the contributions of the imaginary part are particularly small at low values of momentum transfer $\left(q = \frac{2h\nu}{mc^2} \sin\frac{\theta}{2} \right)$. Neglecting the imaginary parts, Bernstein and Mann (1958) have shown by plotting the real parts of the scattering amplitudes that these may be written as,

$$a_{1k} = F_{1k}(q)\,\frac{1+\cos\theta}{2}$$

$$a_{2k} = F_{2k}(q)\,\frac{1-\cos\theta}{2} \qquad \ldots \quad (2)$$

F_{1k} and F_{2k} are smooth functions of q above $q = 0.6$ and for all values of q respectively. Further, F_{2k} is very nearly equal in magnitude to the Bethe's K-shell form factor (Bethe 1952), and consistently large than F_{1k}. Bernstein and Mann (1958) have calculated the contributions of the L shell electrons to the elastic scattering by assuming that the non-spin flip and spin flip amplitudes for L shell contribute in the same ratio as the corresponding K shell amplitudes. However, Brown and Mayers (1957) suggest that this assumption may not be correct at large values of q and experimental data suggest that it is better to neglect the contribution of non-spin flip amplitude for L shell electrons for large values of q So, in our work we have altogether neglected the contributions of non-spin flip amplitudes for L shell electrons at values of q larger than 1. The L shell amplitudes were taken from the work of Bernstein (1958).

The contributions of nuclear Thomson scattering, which interferes constructively with Rayleigh scattering, are calculated from the well known classical relation

Combining the various contributions, Rayleigh plus nuclear Thomson scattering cross section for mercury at a scattering angle θ is given by

$$(d\sigma/d\Omega)_{Ray.+Thom.} = r_0^2/4|([(A^2+B^2)(1+\cos\theta)^2+(C^2+D^2)(1-\cos\theta)^2] \quad \ldots \quad (3)$$

where,

$$A+iB = F_{1k}(q)+F_{1L}(q)+a \qquad \ldots \quad (4)$$
$$C+iD = F_{2k}(q)+F_{2L}(q)+a$$

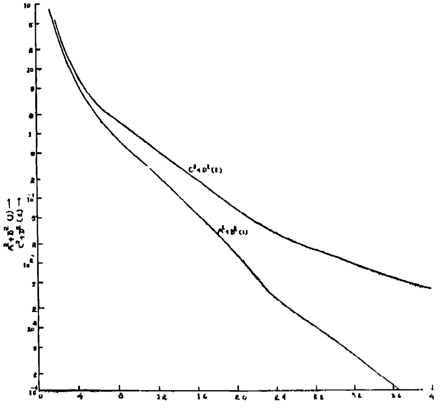
and,

$$a = (Z^2m/M).$$

Here m is the electronic mass, M the nuclear mass and Z the atomic number of the scatterer. $F_{1k}(q)$ and $F_{2k}(q)$ contain both real and imaginary parts as given in the papers of Brown and Mayers (1957), and

$$F_{1k}(q) = (a_{1k}+ib_{1k})/(1+\cos\theta)/2 \qquad \ldots \quad (5)$$
$$F_{2k}(q) = (a_{2k}+ib_{2k})/(1-\cos\theta)/2$$

The values of $a_{1k}+ib_{1k}$ and $a_{2k}+ib_{2k}$ were taken for different scattering angles from the data of Brown and Mayers (1957) for gamma ray energies of 1 28 and 2 56mc^2, and accordingly F_{1k} and F_{2k} calculated as functions of momentum transfer. The values of F_{1k} and F_{2k} were taken from the work of Bernstein (1948) for different values of q The values of A^2+B^2 and C^2+D^2 for mercury, so obtained, are plotted as functions of momentum transfer in Fig. (1). By



Momentum Transfer →

Fig (1) Plots of A^2+B^2 and C^2+D^2 against momentum transfer for mercury. For definitions reference may be made to text

means of these curves, it is possible to estimate the elastic scattering cross sections for mercury at various gamma ray energies and scattering angles

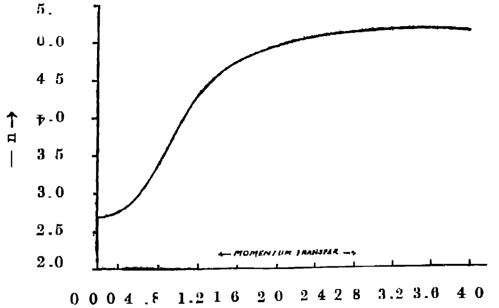


Fig. (2) A Plot of index 'n' of Z dependence against momentum transfer for elastic scattering of gamma rays

The experimental Z dependence curve which shows the variation of index n of Z dependence is given in Fig (2) for various values of momentum transfer (Anand *et al.*, 1964) This curve has been obtained from a study of the variation of the elastic scattering cross sections with atomic number of the scatterer for 0.280, 0.411 and 0 662 MeV gamma rays scattered from Ag, Sn, W and Pb at different scattering angles.

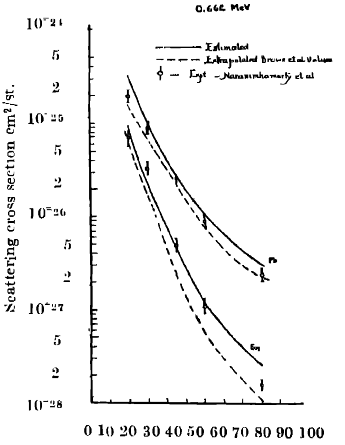
In order to estimate the scattering cross section from the above data for any desired element ($Z \geqslant 47$) at particular values of gamma ray energy and scattering angle, the corresponding cross section for mercury was first evaluated from Fig (1), and eqn (3) To convert this cross section for mercury to the desired element of atomic number Z, the following relation was used,

$$(d\sigma/d\Omega)_{Ray+Thom.}{}^{z,q} = (d\sigma/d\Omega)_{Ray.+Thom}{}^{80,q}(Z/80)^{n} \quad . \quad (6)$$

the index n was obtained from the experimental curve in Fig (2) corresponding to the particular value of q.

RESULTS AND CONCLUSIONS

The estimated cross sections are compared with the available experimental results in Figs (3-6) Figs. (3a, 3b) show a comparison of estimated cross sections for Pb, Pt, Ta and Sn for 0.662 MeV gamma rays at different angles with the experimental data of Narasimhamurty et al. (1964) Also shown in these Figs are



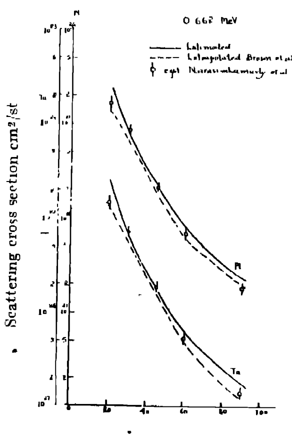
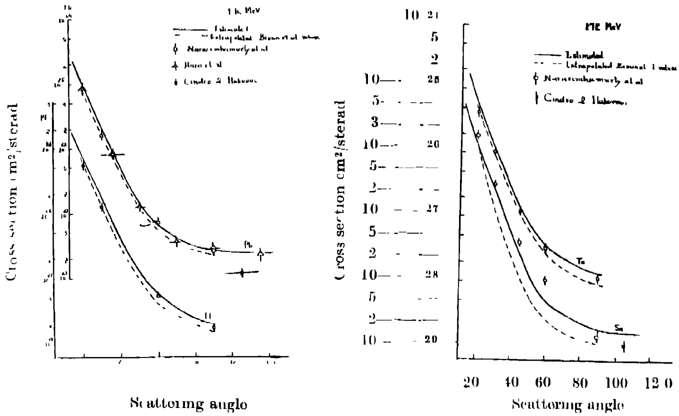


Fig. (3a, 3b) Comparison of the 'estimated' and experimental elastic scattering cross sections for 0.662 MeV gamma rays for Pb, Pt, Ta and Sn.



Figs. (4a, 4b) Comparison of the 'estimated' and experimental elastic scattering cross sections for 1.12 MeV gamma rays for Pb, Pt, Ta and Sn

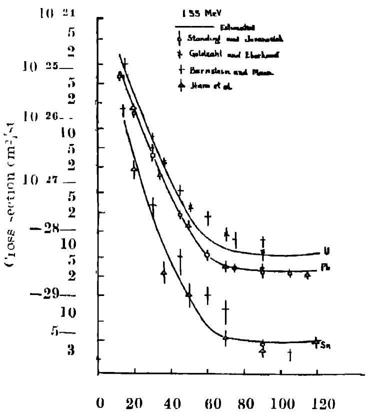


Fig. (5) Comparison of the 'estimated' and experimental elastic scattering cross sections for 1.33 MeV gamma rays for U, Pb, and Sn

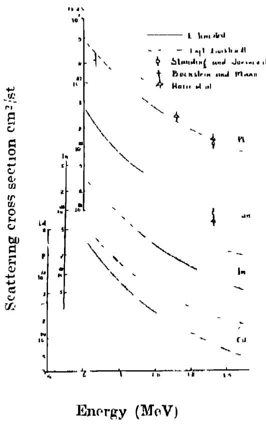


Fig (6) Comparison of 'estimated' and experimental elastic scattering cross sections for Pb, Sn, In and Cd at 90° for different energies.

the extrapolated Brown et al, values of scattering cross sections as reported by Narasimhamurty *et al* (1964). The agreement is better with our estimates. In Figs (4a) and (4b) a comparison of the estimated cross sections for Pb, Pt Ta and Sn at 1.12 MeV is made with the experimental results of Hara *et al.* (1958), Cindro and Ilakovac (1958) and Narasimhamurty *et al.* (1964), and extrapolated theoretical values of calculation of Brown and Mayers (Narasimhamurty *et al.*, 1964). Again the agreement is quite good with the results of Hara *et al.* and Narasimhamurty *et al.* while the results of Cindro and Ilakovac are somewhat lower. The extrapolated values are also lower. Fig (5) compares the estmates at 1.33 MeV for U, Pb and Sn with experimentalvalues of Standing and Jovanovich (1962), Goldzahl and Eberhard (1957), Bornstein and Mann (1958) and Hara *et al.* (1958). The results of Hara *et al.* and Standing and Jovanovich agree well with the estimated values while those of Goldzahl and Eberhard and Bernstein and Mann are appreciably higher. This discrepancy is resolved if, as pointed out by Standing and Jovanovich, the experimental results of Bernstein and Mann are assumed to include some contribution of incoherent scattering. In Fig. (6) we have compared our estimates for Pb, Sn, In and Cd at 90° at different gamma ray energies with the experimental results of Burkhardt (1955) and Hara *et al.* (1958) Here, we find that the experimental data of Burkhardt are consistently higher than the latter more reliable data (Hara *et al*, 1958, Standing and Jovanovich 1962) which are quite scanty This may be due to the presence of appreciable amounts of incoherent scattering in the measurements of Burkhardt (1955).

Thus there exists an over all agreement between the estimated and reliable experimental values of the differential scattering cross sections for different elements at various scattering angles and gamma ray energies. Moreover, the estimates of the scattering cross sections made by Narasimhamurty *et al* (1964) from extrapolation of calculations of Brown *et al* are lower than our estimates and the experimental values. This is particularly noticeable for tin where any deficiency in the method of extrapolation should be clearly exposed

The cross sections of non-resonant elastic scattering estimated by the method described above can be used with confidence to determine the cross section of resonant scattering of gamma rays by comparison with non-resonant scattering cross section, especially when the experimental arrangement needed to restore resonance condition is complicated and it is not possible to calculate the solid angles from the geometrical set up (see for example Moon 1951).

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